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Van Der Waal's Equation of Statein Analysing Viscous Dark Energy Models with the Property of Variable G and Λ

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Abstract

In physical cosmology, cosmological inflation describes a theory of exponential expansion of space in the early universe. The various cosmological features of the model considered are analysed. Inflation explains the origin of the large-scale structure of the cosmos. Here in this paper, a cosmological model is considered with the number of moles (n) together with volume (V) in a gas sample that obeys Van der Wall's equation of state exhibiting the property of variable Cosmological and Gravitational constants (G). The cosmological and gravitational constants and the volume V in the gas sample considered that obeys Van der Wall's equation of state increase exponentially with time whereas the energy density and n, the number of moles in the specified gas sample, decrease exponentially with time. The rate of mass creation during inflation is found to be very large suggesting that all matter in the universe was created during inflation. Inflation is the hypothetical field responsible for exponential expansion i.e., inflation.

Keywords: Dark energy; Evolution; Phantom; Early Universe; Dark matter.

1.0 Introduction

Two proposed forms for dark energy are the Cosmological constant(Λ), a constant energy density filling space homogeneously and scalar fields such as Quintessence or moduli. These are dynamical quantities whose energy density can vary in time and space.

Inflation explains the isotropic nature of the Universe that why it appears to be the same in all directions. Cosmic Microwave Background (CMB) radiation anisotropies observations indicate that the universe can be considered flat. There are several different models proposed to explain the nature of dark energy. The cosmological constant Λ is the simplest model which can be considered here.

The data regarding the present acceleration of the universe as put forward by the Supernova search team can be explained by hypothesizing some exotic matter dominating the present evolution of the Universe that breaks the strong energy condition i.e.,

$$p + 3\rho \geq 0$$

One variant of this exotic matter is the one that violates the null energy condition, viz.,

$$p + \rho > 0$$

The Van der Waal's Equation of State is a thermodynamic equation of state relating the density of gases to the pressure (p), volume (V), and temperature (T). Here, in this paper, Phantom fields are introduced by replacing the pressure p by the effective pressure p_{eff} in the Vander Waal's Equation of State that is

$$p_{eff} = p + b\frac{n^2}{V^2}$$

Viscous effects in an expanding universe are associated with dissipations which are attributed to the creation of energy (matter) in the universe.

2.0 Formulation of the Problem i.e., the Model

Let us consider the Einstein- Hilbert action with a cosmological constant (Λ)

$$S = -\frac{1}{16\pi G}\int d^4 x\sqrt{g}(R + 2\Lambda) + S_{matter} \qquad (1)$$

The variation of the metric with respect to $g_{\mu\nu}$ with

$$f(R) = R - 2\Lambda$$

Gives

$$f^{/}(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} = -8\pi G T_{\mu\nu} \qquad (2)$$

where $T_{\mu\nu}$ is the energy momentum tensor of the cosmic uid. For an ideal fluid, we get,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \qquad (3)$$

whereu_μ, ρ, p are the velocity, density and pressure of the cosmic fluid. Now contracting equation (2), using equation (3) and taking the 00 components of the given equation, we have,

$$Rf^{/}(R) - 2f(R) + 8\pi G T = 0 (4)$$

and $\quad f^{/}(R)R_{00} + \frac{1}{2}f(R) + 8\pi G T_{00} = 0 \quad (5)$

with $\quad T_{00} = \rho, T = \rho - 3p$ and $T_{ij} = -p$ for $i, j = 1,2,3$

For a flat Friedmann- Lemaitre- Robertson Walker Metric,

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2)\right)$$

One should has

$R_{00} = -3\left(\frac{\ddot{a}}{a}\right)$and $R = -6\left(\left(\frac{\dot{a}}{a}\right)^2 + \frac{\dot{a}}{a}\right)$

where a is the scale factor, so that from the equations (4) and (5), we get,

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho + \Lambda \ (6)$$

Also

$$3\left(\frac{\ddot{a}}{a}\right) = -4\pi G(\rho + 3p)\Lambda \ \ (7)$$

The energy momentum equation is given by

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0 \ (8)$$

The pressure p and the energy density ρ of an ideal fluid are related by the equation of state

$p = \omega\rho$, $\omega =$ constant (9)

Considering that Λ and G vary with time for which

$\Lambda = \Lambda(t)$and$G = G(t)$

Considering the Bianchi Identity

$$\left(R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu}\right)_{;\mu} = -(8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu})_{;\mu} = 0 (10)$$

with equations (2) and (3) imply that

$$G\dot{\rho} + 3(\rho + p)G\frac{\dot{a}}{a} + \rho\dot{G} + \frac{\dot{\Lambda}}{8\pi} = 0 \qquad (11)$$

We know that the Vander waal's Equation of State is given as

$$\left(p + b\frac{n^2}{V^2}\right)(V - na) = nRT \qquad (12)$$

where b and c are parameters, they have different values for different gases, where n is the number of moles in a gas sample. V is the volume; R is the universal gas constant. Its value is 0:08206 atm. L/mol K and T is the temperature in Kelvin scale.
In Vander Waal's Equation of State, the effective pressure p is calculated as,

$$p_{eff} = p + b\frac{n^2}{V^2} \qquad (13)$$

Also, from the energy conservation equation

$$\dot{p} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0$$

We have

$$\frac{\dot{a}}{a} = -\frac{\dot{\rho}V^2}{3(pV^2+bn^2+\rho V^2)} \qquad (14)$$

From (13) and (14),

$$G\dot{\rho} + 3\left\{(p+\rho) + b\frac{n^2}{V^2}\right\}.G\frac{\dot{a}}{a} + \rho\dot{G} + \frac{\dot{\Lambda}}{8\pi} = 0$$

Equation (8) gives,

$$p + \rho = -\frac{\dot{\rho}}{3\left(\frac{\dot{a}}{a}\right)}$$

$$\therefore 8\pi\dot{G}\rho + \dot{\Lambda} = -8\pi Gb\left(\frac{\dot{a}}{a}\right)\frac{n^2}{V^2} \qquad (15)$$

Now, we consider the Ansatz,

$$\Lambda = \frac{3\beta}{\rho^{\alpha}} \qquad (16)$$

whereα and β are constants.

Using Equations (15) and (16),

$$8\pi\dot{G}\rho - 3\alpha\beta\frac{1}{\rho^{-(\alpha+1)}}\dot{\rho} = (8\pi G)bH\frac{n^2}{V^2}$$

Therefore,

$$\frac{\dot{G}}{G} - \frac{3\alpha\beta}{8\pi G\rho^{-(\alpha+1)}}\frac{\dot{\rho}}{\rho} = -bH\frac{n^2}{V^2}.\frac{1}{\rho} \qquad (17)$$

where

$$\frac{\dot{a}}{a} = H$$

Since

$$\Lambda = \frac{3\beta}{\rho^{\alpha}}$$

Therefore, from equation (6),

$$H = \frac{1}{\sqrt{3}}\sqrt{(8\pi G\rho - 3\beta\rho^{-\alpha})} \quad (18)$$

From (18) and (17), we get,

$$\frac{\dot{G}}{G} - \frac{3\alpha\beta}{8\pi G\rho^{-(\alpha+1)}}\frac{\dot{\rho}}{\rho} = -\frac{b}{\sqrt{3}}\sqrt{(8\pi G\rho - 3\beta\rho^{-\alpha})}\frac{n^2}{V^2}.\frac{1}{\rho} \qquad (19)$$

Now considering the functional dependence of gravitational constant, we have

$$8\pi G = D\rho^{-(\alpha+1)} \quad (20)$$

where D is a constant.

Using (20) in (19), we get,

$$\frac{\dot{G}}{G} - \frac{3\alpha\beta}{D}.\frac{\dot{\rho}}{\rho} = -\frac{b}{\sqrt{3}}\sqrt{[D\rho^{-\alpha} - 3\beta\rho^{-\alpha}]}.\frac{n^2}{V^2}.\frac{1}{\rho} \quad (21)$$

From (20), we get,

$$G = \frac{D}{8\pi}\rho^{-(\alpha+1)} \quad (22)$$

Also $\dot{G} = -\frac{D}{8\pi}(\alpha+1)\frac{1}{\rho^{(\alpha+2)}}\dot{\rho}$ (23)

Therefore,

$$\frac{\dot{G}}{G} = -(\alpha+1)\frac{\dot{\rho}}{\rho} \quad (24)$$

Substituting the value of $\frac{\dot{G}}{G}$ from (24) in (21), we get,

$$\dot{\rho} = \frac{b(D-3\beta)^{\frac{1}{2}}}{\sqrt{3}[\alpha(1+3\beta)+1]}\rho^{1-\frac{\alpha}{2}}.\frac{n^2}{V^2} \Rightarrow \dot{\rho} = N\rho^{1-\frac{\alpha}{2}}$$

where

$$N = \frac{b(D-3\beta)^{\frac{1}{2}}}{\sqrt{3}[\alpha(1+3\beta)+1]}.\frac{n^2}{V^2} \quad (25)$$

The energy conservation equation (8) becomes

$$\frac{\dot{a}}{a} = -\frac{N}{3(1+\omega)}\rho^{-\frac{\alpha}{2}} \quad (26)$$

Now equation (11) becomes

$$\frac{\dot{\rho}}{\rho} + 3(1+\omega)\left(\frac{\dot{a}}{a}\right) + \frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{D\rho^{-(\alpha+1)}}\frac{1}{\rho} = 0$$

Since $8\pi G = D\rho^{-(\alpha+1)}$

Therefore,

$$\frac{\dot{\rho}}{\rho} + 3(1+\omega)\left(\frac{\dot{a}}{a}\right) - (\alpha+1)\frac{\dot{\rho}}{\rho} - \frac{3\alpha\beta\rho^{-(\alpha+1)}}{D\rho^{-(\alpha+1)}}.\frac{\dot{\rho}}{\rho} = 0$$

$$\Rightarrow A = \rho a^{3(1+\omega)}$$

[Since $\dot{\Lambda} = -3\alpha\beta\rho^{-(\alpha+1)}\dot{\rho}$ and $\frac{\dot{G}}{G} = -\frac{(\alpha+1)\dot{\rho}}{\rho}$ [Equation (14)]]

Also here $A = \left(1 + \frac{3\beta}{D}\right)\alpha$ is another constant. Thus we get,

$$\rho = Aa^{-3(1+\omega)} \quad (27)$$

Now using this value of ρ from (27) in (26), we get,

$$\frac{\dot{a}}{a} = Ka^{\frac{3}{2}(1+\omega)\alpha} \qquad (28)$$

where

$$K = -\frac{NA^{-\frac{\alpha}{2}}}{3(1+\omega)} \text{and} \omega \neq -1$$

Again from equation (28),

$$\log a = Ka^{\frac{3}{2}(1+\omega)\alpha} t + \log(n_1 - \alpha_1)$$

where $\log(n_1 - \alpha_1)$ is a constant of integration, n_1, α_1 both being constants.

i.e.,$a^{-\frac{3}{2}(1+\omega)\alpha}.log\left(\frac{a}{n_1-\alpha_1}\right) = Kt$

Writing $log\left(\frac{a}{n_1-\alpha_1}\right) = \frac{1}{n_1-\alpha_1}$, we get,

$$a^{-\frac{3}{2}(1+\omega)\alpha}.\frac{1}{n_1 - \alpha_1} = Kt$$

$$\therefore a \frac{1}{(n-\alpha)^{-\frac{2}{3(1+\omega)\alpha}}} = -N^{-\frac{2}{3(1+\omega)\alpha}}.A^{-\frac{2}{3(1+\omega)\alpha}}.t^{-\frac{2}{3(1+\omega)\alpha}}$$

Writing the constant term

$$\left[\frac{3(1+\omega)}{(n_1 - \alpha_1)}\right]^{-\frac{2}{3(1+\omega)\alpha}} = \frac{1}{(n-\alpha)^{-\frac{2}{3(1+\omega)\alpha}}}$$

We get,

$$a = Et^{-\frac{2}{3(1+\omega)\alpha}} \quad (29)$$

where $\quad E = A^{\frac{1}{3(1+\omega)\alpha}[-N(n-\alpha)]^{-\frac{2}{3(1+\omega)\alpha}}}, n \neq \alpha$

Substituting this value of a from equation (29) in equation (27), we have,

$$\rho = A_1[-N(n-\alpha)]^{\frac{2}{\alpha}}.t^{2\alpha(1+\omega)^2}, n \neq \alpha \ (30)$$

Writing, $A_1 = A.A^{-\frac{1}{\alpha}} = A^{\frac{\alpha-1}{\alpha}}$, a new constant.

Now the equation (16) becomes,

$$\Lambda = 3\beta A_2[-N(n-\alpha)]^{-2}.t^{-2\alpha^2(1+\omega)^2}, n \neq \alpha (31)$$

Writing $A_1^{-\alpha} = \frac{1}{A_1^{\alpha}} = A_2$, as another constant.

From equation (20),

$$G = \frac{D}{8\pi}A_3[-N(n-\alpha)]^{-2\left(1+\frac{1}{\alpha}\right)}.t^{-2\alpha(\alpha+1)(1+\omega)^2}, n \neq \alpha \ (32)$$

where $A_3 = \frac{1}{A_1^{(\alpha+1)}} = A_1^{-(\alpha+1)}$

3.0 Phantom Energy Evolution

Here we consider the following cases:

3.1. Case (1):

Now let $n = \frac{\alpha^2}{3}, \quad -1 < \alpha < 0 \Rightarrow \alpha\epsilon]-1,0[, 1 + \omega > 0.$

In this case, equations (29) reduces to $a \propto t^{-\frac{2}{3(1+\omega)\alpha}}$

and equation (30) reduces to

$$\rho \propto t^{2\alpha(1+\omega)^2} \ (33)$$

also the equation (32) yield the result

$$G \propto t^{-2\alpha(\alpha+1)(1+\omega)^2}$$

and equation (31) reduces to

$$\Lambda \propto t^{-2\alpha^2(1+\omega)^2} \qquad (34)$$

From equation (20),

$$G = \frac{Da^{3(1+\omega)(\alpha+1)}}{A^{\alpha+1}.8\pi} \text{ where } D > 3\beta > 0 \Rightarrow G > 0$$

Considering the following Equation for a viscous cosmological model,

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho + \Lambda$$

$$\therefore \Lambda \propto H^2 [\text{Since} \Lambda = Aa^{-3(1+\omega)}, \frac{\dot{a}}{a} = H]$$

is equivalent to a viscous dark energy model with

$$\Lambda \propto \rho^{-\alpha} ,$$

Provided $n_{arb} = 1 + \frac{\alpha^2}{3}$, where n_{arb} is the index of viscosity.

3.2 Case (2):

Now let $\alpha > 0$ and $n < \alpha \Rightarrow n - \alpha < 0$ i.e., a negative quantity.

Now the equation $\dot{\rho} = N\rho^{1-\frac{\alpha}{2}}$ together with the two equations (25) and (26), imply that one has the Phantom energy solution, viz, $1 + \omega < 0$. One requires here $D < 0$, so that $G < 0$and for $\beta > 0$, one has $\Lambda > 0$. This solution is found by (Arbab, 2007) and the above solution represents its viscous analogue.

Thus we have found that though the energy density increases, gravity G decreases and viscosity increases.

4.0 Inflationary Solution

We have from Equation (28), when $\alpha = 0$,we have,
$a = \Gamma \exp(Kt)$, Γ =constant (35)

[where $K = -\frac{NA^{-\frac{\alpha}{2}}}{3(1+\omega)}$, $\omega \neq -1$]

where $K > 0 \Rightarrow -\frac{NA^{-\frac{\alpha}{2}}}{3(1+\omega)} > 0 \Rightarrow \frac{NA^{-\frac{\alpha}{2}}}{3(1+\omega)} < 0$.

Two possible cases arise. i.e., either

$NA^{-\frac{\alpha}{2}} < 0$or $3(1 + \omega) < 0 \Rightarrow \omega < -1$.

When $\omega < -1$, it represents Phantom evolution.

Again from equation (27), we have,

$\rho \propto exp[-3K(1 + \omega)t]$(36)

Again from the equation (16) we have,

$\Lambda \propto exp[3\alpha K(1 + \omega)t]$ (37)

Next from equation (20), we have,

$G \propto exp[3K(1 + \omega)(1 + \alpha)t]$(38)

If in the Van der Waal's Equation of State,

$$\left(p + b\frac{n^2}{V^2}\right)(V - na) = nRT$$

we take the parameters a and b as zero, then we shall arrive at the reduced form of Van der Waal's Equation, which is analogous to the Classical Ideal Gas Law and thus we shall have the equation as

$f(p, V, T) = pV - nRT = 0$ (39)

$\Rightarrow f(p, V, T) = [\rho(\gamma - 1)e]V = nRT$ (40)

$\Rightarrow f(p, V, T) = n = \frac{[\rho(\gamma-1)e]V}{RT}$ (41)

Also we can have,

$f(p, V, T) = V = \frac{nRT}{[\rho(\gamma-1)e]V}$ (42)

This can also be further expressed as

$p = \rho(\gamma - 1)e$ (43)
where $\gamma = \frac{C_p}{C_v}$ is the adiabatic index i.e., ratio of specific heats.
$e = C_v T$
is the internal energy per unit mass i.e., the specific internal energy, C_v is the specific heat at constant volume and $C_p$is the specific heat at constant pressure.

Now from equation (41), we have,

$n \propto exp[-3K(1 + \omega)t]$ (44)
since$\rho = Aa^{-3(1+\omega)}$

Again from equation (42), we have,
$V \propto exp[3K(1 + \omega)t]$ (45)

Here it is noticed that during inflation
$\omega \neq -1$

From equation (28), we get,
$\omega \neq -1$

This is unlike the special case, where inflation requires
$\omega = -1$

The large decrease of the bulk viscosity during the inflationary era has allowed the universe to isotropize and eventually led to the isotropic and homogeneous universe, as we observe today.

Now from equation (35) and (36), we notice that the mass created (annihilated), during inflation is
$M \propto exp(-3K\omega t)$
But since $\omega > -1$, one has for $-1 < \omega < 0$, a positive mass creation rate. Hence, one should presume that all matter constituting the universe mass was produced during decelerated inflation.

5.0 Discussion

Inflation is induced by dark energy only. Thus, dark energy played an important role by driving the early universe into an exponential expansion and thus the present universe into an accelerated expansion.

Here, in this paper, the effect of the number of moles (n) in a gas sample and volume (V) in the evolution of dark matter and phantom cosmologies. The increasing (n) and the volume (V) in the gas sample that obeys Van der Waal's equation of state and the decreasing gravitational constant do not allow the phantom energy density to condensate. During inflation, the Cosmological constant increases exponentially with time, whereas the energy density decreases exponentially. Before inflation, whatever may be the initial value of the Cosmological constant; its value will be enormously large, at the end of the inflation. During inflationary era, the universe isotropizes and the Cosmological constant (Λ) attains a very large value. After inflation, the Cosmological constant decreases with time. This evolution provides a viable mechanism for the smallness of the present Cosmological constant that is why the Cosmological constant is vanishingly small as compared to its initial value. Dark energy has been used as a crucial ingredient in a recent attempt to formulate a cyclic model for the universe.

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