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A Universe with Different Manifestations of Dark Energy

Kangujam Priyokumar Singh[1] and Pallavi Saikia[2*]

[1]Department of Mathematical Sciences, Bodoland University, Kokrajhar, Assam, India
[2]Department of Mathematics, Kokrajhar Govt. College, Kokrajhar, Assam, India

***Corresponding Author:** Pallavi Saikia, Assistant Professor, Dept. of Mathematics, Kokrajhar Govt. College, Kokrajhar, Assam, India; Tel: 09435515283; E-mail: pallavisaikia_kok@rediffmail.com

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Abstract

In Physical Cosmology and Astronomy, dark energy is regarded as the hypothetical form of energy which is extremely responsible for the exponential expansion of the universe. The standard cosmological model indicates that the total mass-energy of the universe contains 4.9% ordinary matter, 26.8% dark matter and 68.3% dark energy.In 1998, published observations of Type Ia supernovae by the High-Z Supernova Search Team followed by the Supernova Cosmology Project in 1999 suggested that the expansion of the universe is accelerating. The existence of dark energy is needed to reconcile the measured geometry of space with the total amount of matter in the universe. Measurements of cosmic microwave background (CMB) anisotropies indicate that the universe is close to flat. There are two proposed forms for dark energy, cosmological constant and scalar fields such as quintessence. In this paper, we have considered a model universe and have studied the overall phenomenon of the universe, i.e., the formation of dark energy as well as dark matter. Here, we have studied the anisotropy parameter of the model universe to arrive at a conclusion that whether our assumed model is an isotropic one or not. Also we have calculated the pressure and density of the model universe to examine the accelerated expansion of the universe. Here, the model universe we have considered is assumed to follow spatially flat Friedmann Robertson Walker (FRW) Metric, where in the Chaplygin gas model; we have applied the equation of state of the barotropic fluid, which explains the formation of dark matter preceding the formation of dark energy which is extremely responsible for the exponential expansion of the universe.

Keywords: Model Universe; Dark Energy; Dark Matter; Density; Pressure.

1. Introduction

The model under consideration leads to the fact that the scalar expansionθ is an increasing function of time. The asymptotic behavior ρ of the modeluniverse leads us to a clear distinction of the early universe and the lateuniverse based on the property that when$a \leq 1$, we find

$\rho \sim (1+\omega_0) a_0^{3(1+\beta)} a^{-3}$

and in the late universe, that is, when $a \geq 1$, we find

$\rho \sim (-\omega_0)^{\frac{1}{1+\beta}}$

These results imply that in the early Universe, the energy density behavesas $\rho \propto a^{-3}$, which is the same as in the case of non-relativistic matter suchas dark matter.
Again in the case of the late universe, the energy density behaves in theform

$\rho \to (-\omega_0)^{\frac{1}{1+\beta}}$which shows that the energy density tends to a constant which means that it can play the role of dark energy. Thus, it is seen that the generalizedChaplygin gas model can explain the origin of dark energy as well as darkmatter simultaneously.

2. Formulation of the Problem i.e., the Model

Here in this paper, we consider the specifically flat Friedmann RobertsonWalker (FRW) model, having the metric

$$ds^2 = dt^2 - a^2 t[dr^2 + r^2 d\theta^2 + r^2 sin^2\theta \, d\phi^2](1)$$

The energy momentum tensor is given by

$$T_{\mu\nu} = e u_i u_j + p(u_i u_j - g_{ij}) \, (2)$$

where $p =$ fluid pressure
$\rho =$fluid density
$u_i =$the four flow vector satisfying the relation

$$g_{ij} u^i u^j = 1 \, (3)$$

The Einstein field equation is given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G[\rho u_i u_j + p(u_i u_j - g_{ij})](4)$$

From which we have,

$$8\pi G\rho = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}(5)$$

and $$8\pi G\rho = 3\frac{\dot{a}^2}{a^2}(6)$$

The equation of state for Chaplygin gas can be written as

$$p = -\frac{A}{\rho^{\frac{1}{z}}}(7)$$

where $1 \le z < \infty$

On the other hand, the equation of state for the barotropic fluid is givenby

$$p = \omega\rho(8)$$

with $\omega < -1$

Thus from equations (7) and (8) we have,

$$\omega = -\frac{A}{\rho^{\left(\frac{z+1}{z}\right)}}(9)$$

The energy momentum equation is given by

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + p) = 0$$

$$\Rightarrow \dot{\rho} = -3\left(\frac{\dot{a}}{a}\right)(\rho + p)(10)$$

Using equations (7) and (10), we get,

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left[\rho - -\frac{A}{\rho^{\frac{1}{z}}}\right](11)$$

On integrating equation (11) we have,

$$\rho(t)^{\frac{1+z}{z}} = -3\left[\rho^{\frac{1+z}{z}} - A\right]\log a + A$$

where A is the constant of integration andρ is a function of time, since $\rho = \rho(t)$

$$\therefore \quad \rho(t)^{\frac{1+z}{z}} = A + \left[\rho^{\frac{1+z}{z}} - A\right] log\, a^{-3}(12)$$

Now writing,

$$log\, a^{-3} = \left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}$$

Equation (12) becomes

$$\rho(t)^{\frac{1+z}{z}} = A + \left[\rho_p^{\frac{1+z}{z}} - A\right]\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)} (13)$$

whereρ_p and a_p are the values of ρ and a at present time t_p

Now from equation (9), we have,

$$A = -\omega_P \rho_P^{\left(\frac{1+z}{z}\right)} (14)$$

where$\omega_p = \omega(t_p)$, t_p being the present time.

Now substituting the value of A from equation (14) in equation (13), we have,

$$\rho = \rho_p \left[-\omega_p + (1 + \omega_p)\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}\right]^{\left(\frac{z}{1+z}\right)} (15)$$

One of the well-studied dark energy models is the Quintessence model ([1] and [2]), which is ascalar field model described by a field ϕ and a $V(\phi)$ potential. It represents the simplestscalar-field scenario without having theoretical problems such as the appearance of ghostsand Laplacian instabilities. The energy density ρ_Q and the pressure $P_Q$of the quintessencescalar field model are given, respectively, by

$$\rho_Q = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$P_Q = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

In the homogeneous model of the universe, the above two equations can be expressed as

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \, (16)$$

and $$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)(17)$$

where $V(\phi)$is the potential of the scalar field $\phi(t)$.

From the equations (7) and(14), we have,

$$\frac{\dot{\phi}^2}{2} = \frac{\omega_p \rho_p^{\left(\frac{1+z}{z}\right)}}{\rho^{\frac{1}{z}}} + V(\phi)$$

Again from equations (7) and (16), we have,

$$\dot{\phi}^2 = \frac{\rho^{\left(\frac{1+z}{z}\right)} + \rho_p^{\left(\frac{1+z}{z}\right)} \omega_p}{\rho^{\frac{1}{z}}} (18)$$

Now substituting the value of ρ from equation (15) into the equation (18),we get,

$$\dot{\phi}^2 = \frac{(1+\omega_p)\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)} . \rho_P}{\left[-\omega_p + (1+\omega_p)\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}\right]^{\frac{1}{1+z}}} \quad (19)$$

Now for $\omega_p > -1$, we see that $\dot{\phi}^2 > 0$, that gives positive kinetic energy.Thus, in this case, the form of the dark energy will be quintessence. Again,for$\omega < -1$, we see that$\dot{\phi}^2 < 0$, in which case, the dark energy is negative.Thereby the form of dark energy in this case will be phantom. Thus, thescalar field we have taken manifests the three different forms of dark energyviz., the generalized Chaplygin gas, Quintessence and Phantom energy.

In this case of the form of dark energy, the Einstein Friedmann equation gives

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} . \rho$$

$$\Rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} . \rho_p \left[-\omega_p + (1+\omega_p)\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}\right]^{\left(\frac{z}{1+z}\right)} \quad (20)$$

$$\Rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} . \Omega_p . \rho_p^{(c)} \left[-\omega_p + (1+\omega_p)\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}\right]^{\left(\frac{z}{1+z}\right)} \quad (21)$$

where $\Omega_p = \frac{\rho_P}{\rho_p^{(c)}}$(22)

Here$|\omega_p| > 1$ and $\rho_p^{(c)} = \frac{3H_p^2}{8\pi G_N}$ (23)

where G_N being the Newtonian Gravitational Constant.

From equation (23), we have,

$$H_p^2 = \frac{8\pi G_N \rho_p^{(c)}}{3} \quad (24)$$

Here H_p is the present value of the Hubble constant.

Now substituting these values in equation (21), we have,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} . \Omega_p . \rho_p^{(c)} \left[|\omega_p| + (1+|\omega_p|)\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}\right]^{\left(\frac{z}{1+z}\right)}$$

$$\Rightarrow \frac{\dot{a}}{a} = H_p . \Omega_p^{\frac{1}{2}} \left[|\omega_p| + (1+|\omega_p|)\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}\right]^{\frac{z}{2(1+z)}} \quad (25)$$

Now expanding the R.H.S. of the above equation up to second order terms and neglecting higher order terms, we have,

$$\frac{\dot{a}}{a} \simeq H_p . \Omega_p^{\frac{1}{2}} |\omega_p|^{\frac{z}{2(1+z)}} \left[1 + \frac{z(1-|\omega_p|)}{2(1+z)|\omega_p|}\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}\right] \quad (26)$$

Integrating equation (26) we have,

$$a(t) = \frac{a_p}{\left[2(1+z)|\omega_p|\right]^{\frac{z}{3(1+z)}}} \left[z + 2(1+z)|\omega_p| e^{6H_p|\omega_p|^{\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}}} - z(1-|\omega_p|)\right]^{\frac{z}{3(1+z)}} \quad (27)$$

$$\Rightarrow \frac{a_p}{a(t)} = \frac{[2(1+z)|\omega_p|]^{\frac{z}{3(1+z)}}}{\left[z+2(1+z)|\omega_p|e^{6H_p|\omega_p|\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}}-z(1-|\omega_p|)\right]^{\frac{z}{3(1+z)}}} \quad (28)$$

Now substituting this value of $\frac{a_p}{a(t)}$ from equation (28) into the equation (15)

i.e., $\rho = \rho_p \left[-\omega_p + (1+\omega_p)\left[\frac{a_p}{a(t)}\right]^{3\left(\frac{1+z}{z}\right)}\right]^{\left(\frac{z}{1+z}\right)}$ we have,

$$p = \rho_p \left[-\omega_p + (1+\omega_p)\left[\{2(1+z)|\omega_p|\}.\left[z+2(1+z)|\omega_p|e^{6H_p|\omega_p|\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}}-z(1-|\omega_p|)\right]^{-1}\right]^{\left(\frac{z}{1+z}\right)}\right] \quad (28)$$

Now from equations (7) and (14), we have

$$p = \frac{\omega_p \rho_p^{\frac{(1+z)}{z}}}{\rho^{\frac{1}{z}}}$$

$$\Rightarrow p = \omega_p \rho_p^{\frac{(1+z)}{z}} \rho_p \left[-\omega_p + (1+\omega_p)\left[\{2(1+z)|\omega_p|\}.\left[z+2(1+z)|\omega_p|e^{6H_p|\omega_p|\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}}-z(1-|\omega_p|)\right]^{-1}\right]^{\left(\frac{z}{1+z}\right)}\right] \quad (29)$$

Thus, volume

$V = a^3$is

$$V = \frac{a_p^3}{[2(1+z)|\omega_p|]^{\left(\frac{z}{1+z}\right)}}\left[z+2(1+z)|\omega_p|e^{6H_p|\omega_p|\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}}-z(1-|\omega_p|)\right]^{\left(\frac{z}{1+z}\right)} \quad (30)$$

[Substituting the value of $a(t)$from equation (26)]

From the equation (24) i.e., $H_p^2 = \frac{8\pi G_N \rho_p^{(c)}}{3}$ we have,

$$H = \left(\frac{8\pi G_N}{3}\right)^{\frac{1}{2}} \rho_P^{\frac{1}{2}} \left[-\omega_p + (1+\omega_p)\left[\{2(1+z)|\omega_p|\}.\left[z+2(1+z)|\omega_p|e^{6H_p|\omega_p|\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}}-z(1-|\omega_p|)\right]^{-1}\right]^{\left(\frac{z}{1+z}\right)}\right] \quad (31)$$

Shear stress,$\sigma = 0$(32)

Anisotropy parameter $\Delta = 0$(33)

We have, $H_x = H_y = H_z = \frac{\theta}{3} = \left(\frac{8\pi G_N}{3}\right)^{\frac{1}{2}} \rho_P^{\frac{1}{2}} \left[-\omega_p + (1+\omega_p)\left[\{2(1+z)|\omega_p|\}.\left[z+2(1+z)|\omega_p|e^{6H_p|\omega_p|\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}} - z(1-|\omega_p|)\right]^{-1}\right]^{\left(\frac{z}{1+z}\right)}\right]$

Also $\theta = \frac{\dot{V}}{V} = 3\frac{\dot{a}}{a}$

$$\therefore \theta = 3\frac{\theta}{3} = 3.\left(\frac{8\pi G_N}{3}\right)^{\frac{1}{2}} \rho_P{}^{\frac{1}{2}} \left[-\omega_p + (1+\omega_p)\left[\{2(1+z)|\omega_p|\}.\left[z + 2(1+z)|\omega_p|e^{6H_p|\omega_p|^{\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}}} - z(1-|\omega_p|)\right]^{-1}\right]^{\left(\frac{z}{1+z}\right)}\right] \quad (34)$$

Also $\because q = -1 - \dot{H}H^{-2}$

$$\therefore q = -1 - \frac{1}{2}\left(\frac{8\pi G_N}{3}\right)^{-\frac{1}{2}} \rho_P{}^{-\frac{3}{2}} \left[-\omega_p + (1+\omega_p)\left[\{2(1+z)|\omega_p|\}.\left[z + 2(1+z)|\omega_p|e^{6H_p|\omega_p|^{\frac{z\sqrt{\Omega_p(t-t_p)}}{2(1+z)}}} - z(1-|\omega_p|)\right]^{-1}\right]^{\frac{-3\left(\frac{1+z}{z}\right)}{2}}\right] \quad (35)$$

3. Discussion

The volume and the scale factor of the Universe are found to increase rapidly with time. Thus complying with the accelerated expansion of the Universe, the anisotropy parameter of expansion of this model is seen to bezero which shows that our Universe is almost isotropic. As almost the pressure of this model Universe is negative, it helps in the accelerated expansion of the Universe, because a negative pressure stimulates a repulsive gravity and inflates the Universe by overwhelming the usual gravitational effects of matter. In this model, the directional Hubble parameter, anisotropy parameter and shear tensor come out to be constants. Here it is seen that the equation of state is found to be negative, but a decreasing function of time. Thus our Universe starts as a Universe, first containing the dark energy in the form of Chaplygin gas, then the dark energy taking the form of Quintessence and finally our model becomes a Universe filled with dark energy in the form of Phantom Energy [3, 4].

At time t_s given by

$$(z + 2(1+z)|\omega_0|)e^{6H_p|\omega_p|^{\frac{z}{2(1+z)}}.\sqrt{\Omega_0}(t-t_0)} = z(1-|\omega_0|)^{\frac{z}{3(1+z)}}$$

There is a singularity or a bounce, perhaps which an instant of changing is from one form of dark energy to another form. At other times, there is an accelerated expansion as $a(t)$ tends to infinity as t tends to ∞.Here thescalar expansion θ is found to be an accelerated expansion of the Universe.

On the other hand, the pressure and density of this model are found to have finite values at infinite time. As also the anisotropy parameter of this universe is found to be zero, we can see that our model is an isotropic one. Moreover, from the asymptotic behaviour of ρ of the model universe, as wehave obtained here, we can conclude as under. In the early universe, that is,when $a \leq 1$, we find that

$\rho \sim (1+\omega_0)a_0{}^{3(1+\beta)}a^{-3}$

and in the late universe, that is, when $a \geq 1$, we have found that

$\rho \sim (-\omega_0)^{\frac{1}{1+\beta}}$

These results imply that in the early Universe, the energy density behaves as $\rho \sim a^{-3}$, which is the same as in the case of non-relativistic matter suchas dark matter. Again in the case of the late universe, the energy densitybehaves in the form

$\rho \to (-\omega_0)^{\frac{1}{1+\beta}}$

which shows that the energy density tends to a constant which means that it can play the role of dark energy. Thus, it is seen that the generalized Chaplygin gas model can explain the origin of dark energy as well as dark matter simultaneously and also the formation of dark matter preceding the formation of dark energy [3 - 5]. Thus, taking a single fluid in the form of generalized Chaplygingas, both the origin of dark matter and dark energy can be defined and later on this fluid behaves as Quintessence and Phantom forms of dark energy.

4. References

[1] Ratra B and Peebles PJE, Phys. Rev. D 37, 3406 (1988).

[2] Khurshudyan M., Chubaryan E., Pourhassan B.,Interacting Quintessence Models of Dark Energy, International Journal of Theoretical Physics [arXiv: 1402.2385 [gr-qc]].

[3]Pavek R., "Dark Energy: The Biggest Mystery of the universe" National Geographic News, (2011).

[4]Bertone G, "Phys of the Dark Universe" Frontiers in Particle Astrophysics and Cosmology, Elsevier.

[5]Baudis L. "Direct Dark Matter Detection: the Next Decade" Dark Universe, p. 94-108, (2012).

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