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On Einstein's Program and Maxwell's Equations

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Abstract

The Einstein's programforms a consistent system for universe description, beside the standard model of particles. It is founded upon a scalar field propagating at speed of light c, which constitutes a common relativist framework for classical and quantum properties of matter and interactions. Matter corresponds to standing waves. Classical domain corresponds to geometrical optics approximation, when frequencies are infinitely high. A space-like function $u_0(k_0r_0)$, reducing to a Dirac's distribution $\delta(r_0)$, describes a massive particle. Quantum domain corresponds to wave optics approximation. Adiabatic variations of frequencies lead to electromagnetic interaction constituted by progressive waves, to dynamical law of energy-momentum conservation, and to least action principle. The Einstein's program allows, in particular, to exhibit implicit contents of Maxwell's equations, in wave-particle duality, in dynamical properties of matter, and in quantification following adiabatic variations of frequencies.

Keywords: Einstein's program; Maxwell's equations; Adiabatic invariant; Wave-particle duality

1. Introduction

It is well known that the Maxwell's equations plaid a fundamental role at origin of special relativity by Einstein [1], yielding the two main independent and separated branches of physics. The first one is classical and determinist. It led to general relativity, which describes gravitation, as acting theoretically and universally upon matter and all interactions. The other is probabilistic quantum mechanics. It led to the Standard Model of particles, with the whole universe composed of fundamental particles, both for matter and for three, out of the four known interactions. All manifest experimentally either as waves or as particles.

Until now, in spite of many important works, gravitation has resisted to quantification. Inversely, the probabilistic quantum approach has resisted to its derivation from a unitary classical field, following Einstein's works, which recognized that, beyond his disagreement, its formalism validity should be maintained.

«*The statistical character of the present theory would then have to be a necessary consequence of the incompleteness of the description of the systems in quantum mechanics…*
Above all, however, the reader should be convinced that I fully recognize the very important progress which the statistical quantum theory has brought in physics.... this theory is until now the only one which unites the corpuscular and undulatory dual character of matter in a logically satisfactory fashion; and the (testable) relations, which are contained in it, are, within the natural limits fixed by the undeterminacy-relation, complete. The formal relations which are given in this theory—i.e., its entire mathematical formalism—will probably have to be maintained, in the form of logical inferences, in every useful future theory."[2]

For that purpose, he left a program of research [3], founded upon a general energy field.

«*We have two realities: matter and field. ….We cannot build physics on the basis of the matter concept alone. But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build pure field physics? We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created….only field-energy would be left, and the particle would be merely an area of special density of field-energy. In that case one could hope to deduce the concept of the mass-point together with the equations of the motion of the particles from the field equations- the disturbing dualism would have been removed... One would be compelled to demand that the particles themselves would everywhere be describable as singularity free solutions of the completed field-equations….One could believe that it would be possible to find a new and secure foundation for all physics upon the path which had been so successfully begun by Faraday and Maxwell*.»

From an experimental point of view, the Einstein's Program has been validated by the International Legal Metrology Organization when it admitted, in one hand, that the light velocity in vacuum was a primary fundamental constant in physics, with its numerical value strictly fixed, and in other hand, when it adopted one particular period an electromagnetic wave frequency as standard for measures of time [4].

From a theoretical point of view, the Standard Model derives all particles from relativistic quantum fields: with specific mass quarks and leptons, and with mass-less c-fields for photons and gluons. The classification of particles as fermions or bosons emphasizes the lasting role of Einstein in quantum theory, since it leans upon Bose-Einstein or Fermi-Dirac statistics.

Then Einstein's works, present when the paths of two branches of physics, classical relativist and quantum probabilistic diverged, still hold nowadays with the separated General Relativity and Standard Model.

As shown in different articles [5-9], the Einstein's Program forms a consistent system for universe description. It is founded upon a scalar field propagating at speed of light c. Matter properties derive from standing waves, and electromagnetic interaction from adiabatic variations of frequencies. In the geometrical optics approximation, when very high frequencies are experimentally undetectable, the oscillations are hidden. This holds at one and the same time in classical relativistic and quantum frame works, yielding both descriptions to be incomplete.

Beside the Standard Model, the Einstein's Program enlarges our approach, comparably to using both eyes for tridimensional vision, or both ears for stereophonic audition.

In this article we propose to show how the Einstein's program offers means for a renewed approach of Maxwell's equations. It allows, in particular, exhibiting its implicit contents in field-particle duality, in dynamical properties of matter, and in quantification following adiabatic variations of frequencies.

2. The Einstein's Program

We restrict to summarize some equations deduced from Einstein's program [5-9], in order to show how they are related to main equations of Maxwell's equations and to quantum mechanics, otherwise widely documented.

2.1 Kinematical Properties of Standing Fields

From the d'Alembertian's equation describing a scalar field ε propagating at light velocity c

$$\Box\varepsilon = \Delta\varepsilon - (1/c^2)(\partial^2\varepsilon/\partial t^2) = 0, \quad \partial^\mu\partial_\mu\varepsilon = 0 \quad (1)$$

Derive two kinds of elementary harmonic solutions with constant frequency ω_0, with different kinematical properties. The progressive waves, retarded $\cos(\omega_0 t_0 - k_0 x_0)$ or advanced $\cos(\omega_0 t_0 + k_0 x_0)$, are in motion with velocityof lightc=ω_0/k_0. The standing waves $\varepsilon_0(x_0,t_0) = u_0(k_0x_0)\psi_0(\omega_0 t_0) = \cos(\omega_0 t_0)\cos(k_0 x_0)$, where space and time variables are separated, oscillate locally. They allow defining a system of coordinates at rest (x_0,t_0).

They may be considered as resulting from superposition of progressive waves

$$\cos(\omega_0 t_0 + k_0 x_0) + \cos(\omega_0 t_0 - k_0 x_0) = 2\cos(\omega_0 t_0)\cos(k_0 x_0). \quad (2)$$

When, in a system of reference (x,t), the frequencies of opposite progressive waves are different

$$\cos(\omega_1 t - k_1 x) + \cos(\omega_2 t + k_2 x) = 2\cos(\omega t - \beta k x)\cos(kx - \beta\omega t), \quad (3)$$

Where $\omega = (\omega_1 + \omega_2)/2 = kc$, and $\beta = (\omega_1 - \omega_2/\omega_1 + \omega_2)$. By identification with (2), they form a standing wave in motion with a speed $v = \beta c = (\omega_1 - \omega_2/\omega_1 + \omega_2)c$ with frequency $\omega = (\omega_1 + \omega_2)/2 = kc$, and $\omega_0 = \sqrt{\omega_1\omega_2}$ at rest, defining the Lorentz transformation between the systems of reference(x_0,t_0) and (x,t), and leading to its whole consequences.

It can be shown that the Lorentz transformation, fundamental in special relativity, is specific of c-field standing waves, particularly through the coefficient $\sqrt{(1-\beta^2)}$, which becomes $(1\pm\beta)$ for progressive waves [6].The four-dimensional Minkowski's formalism expresses invariance properties of standing waves at rest, in which the variables of space and time are separated, when they move uniformly with a speed $v = \beta c < c$ since β is a relative difference. Confirmation is found into invariant quantities obtained from four-quantities, such as coordinates $x_\mu x^\mu = x_0^2$ or $x_\mu x^\mu = c^2 t_0^2$, and functions $u_\mu u^\mu = u^2(x_0)$ or $\psi_\mu\psi^\mu = \psi^2(t_0)$. Their space-like or time-like characters are absolute, depending of their refering quantities defined in the rest system.

Since the functions $u_0(k_0x_0)$ and $\psi_0(\omega_0t_0)$ are independent, the frequency ω_0 is necessarily constant in

$$(1/u_0)\Delta_0u_0 = (1/\psi_0)(\partial^2\psi_0/c^2\partial t_0^2) = -k_0^2 = -\omega_0^2/c^2 \quad (4)$$

The function of space $u_0(k_0x_0)$, obeying the Helmholtz's equation at rest $\Delta_0u_0 + k_0^2u_0 = 0$, which becomes in motion $\Delta u - (1/c^2)(\partial^2u/\partial t^2) + k_0^2u = 0$. It describes geometrical properties of standing waves. It verifies Bessel spherical functions solutions, and particularly its simplest elementary solution, with spherical symmetry, finite at origin of the reference system,

$$u_0(k_0r_0) = (\sin k_0r_0)/(k_0r_0) \quad (5)$$

It represents a lumped function. In geometrical optics approximation, when the frequency is very high, tending towards infinity $\omega_0 = k_0c \rightarrow \infty$, the space function tends towards Dirac's distribution $u_0(k_0r_0) \rightarrow \delta(r_0)$. The standing wave of the field behaves as a free classical material particle isolated in space [6].

From a kinematical point of view, the central extremum of an extended standing wave, either at rest or in motion, is appropriate to determine its position $x_0 = r_0 = 0$ in cartesian system of reference, exactly like the centre of mass in mechanics. It verifies, for instance from (5),

$$\boldsymbol{\nabla_0} u_0(\mathbf{x}_0) = 0 \quad (6)$$

In order to point out the constant frequency of a standing field, we express it as

$$\varepsilon(\omega t, kx) = u(kx, \beta\omega t)\exp i(\omega t - \beta kx) \quad \varphi = \omega t - \beta kx \quad (7)$$

The equations of special relativity are based on mass-points, as singularities, moving on trajectories, deriving then directly from geometrical optics approximation. The periodic equations, generic of standing fields, are hidden. The space coordinates x_α, involved in the space-time metric, are point-like dynamical variables, and not field variables r, which would describe an extended amplitude repartition in space. Then, the kinematic properties of standing waves for a scalar field propagating at light velocity c, with constant frequency ω and velocity v, reduce formally to kinematical properties of isolated point-like matter.

2.2 Dynamical Properties of Standing Fields

Since the field $\varepsilon(\omega t, kx)$ cannot be infinite with respect to space and time, one limits it by imposing boundary conditions. They are usually exerted by matter. It behaves either as a source which fixes the frequency ω, or as a detector annealing it, or also as a geometrical space boundary fixing the wavelength λ through $k = 2\pi/\lambda$. This is not felicitous from relativistic consistency, since space and time operate separately. What is more, matter is heterogeneous with respect to field. In order to remain in homogeneous frame, we rather consider boundaries provided by wave packets. Two progressive waves with different frequencies ω_1, ω_2 propagating in the same direction at light velocity, give rise to a wave packet propagating in the same direction at light velocity. Its main wave with frequency $\omega = (\omega_1 + \omega_2)/2$, is modulated by a wave with frequency $\beta\omega = (\omega_1 - \omega_2)/2 = \Delta\omega/2 = \Delta kc/2$, wavelength $\Lambda = 2\pi/\beta k$, and period $T = \Lambda/c$. Since $\beta < 1$, the modulation wave acts as an envelope with space and time extensions $\Delta x = \Lambda/2, \Delta t = T/2$, yielding well known Fourier relations $\Delta x.\Delta k = 2\pi$ and $\Delta t.\Delta\omega = 2\pi$.

Then, Fourier relations represent homogeneous relativist boundary conditions for the scalar field ε. From a physical point of view, they must necessarily supplement the d'Alembertian's equation (1) in order to emphasize that the field cannot extend to infinity with respect to space and time.

When the difference of frequencies $\beta\omega = (\omega_1 - \omega_2)/2 = \Delta\omega/2 << \omega$ is very small, it can be considered as a perturbation with respect to the main frequency, $\beta\omega = \delta\omega$. Then a wave packet can be assimilated to a progressive monochromatic wave with frequency $\Omega = \omega \pm \delta\omega$, inside the limits fixed by the component frequencies $\omega_1 = \omega + \delta\omega$ and $\omega_2 = \omega - \delta\omega$. By difference with standing waves frequencies, which must be constant and monochromatic, progressive field's solutions of (1), may be more complex, with frequencies varying with space and time. We will characterize an almost monochromatic wave by a frequency $\Omega(x,t)$, varying very little around a constant ω

$$\Omega(x,t) = K(x,t)c = \omega \pm \delta\Omega(x,t) \quad \delta\Omega(x,t) << \omega \quad \omega = \text{constant}. \quad (8)$$

We recognize the definition of an adiabatic variation for the frequency [10]. Consequently, all following properties of almost fields arise inside such a process. The necessarily constant frequency of a standing wave must be considered, not as a given data, but rather as the mean value, all over the field, of different varying frequencies $\Omega(x,t)$. The perturbation frequencies $\delta\Omega(x,t)$ of modulation waves propagating at light velocity, behave as interactions between main waves, which yield the mean frequency ω to remain practically constant all over the space-time.

Such a behavior authorizes mathematically to derive almost fields properties from monochromatic ones, through the variation of constants method (Duhamel principle). Instead of (7), we express it, as

$$\varepsilon(x,t) = U(x,t)\exp i\phi(x,t) \quad \phi(x,t) = \Omega(x,t)t - \mathbf{K}(x,t).\mathbf{x} + 2n\pi \quad (9)$$

It leads to

$$d\phi(x,t) = \Omega(x,t)dt - \mathbf{K}(x,t).d\mathbf{x} \approx \omega dt - \mathbf{k}.d\mathbf{x}. \quad U(x,t) = u(x,t) \pm \delta U(x,t) \quad (10)$$

Where products of second order $\delta\Omega dt\approx 0$ and $\delta\mathbf{K}.d\mathbf{x}\approx 0$, defined modulo 2π, are neglected at first order of approximation. This is equivalent to incorporate directly the boundary conditions defined by Fourier relations, in almost monochromatic solutions.

Following (1), the field $\varepsilon(x,t)$ defined by (9) verifies,

$\partial^\mu\partial_\mu U - U\partial^\mu\phi\partial_\mu\phi = 0$

or

$\partial^2U/c^2\partial t^2 - \nabla^2U - U[(\partial\phi/c\partial t)^2 - (\nabla\phi)^2] = 0$ (11)

$\partial^\mu(U^2\partial_\mu\phi) = 0$

or

$\partial(U^2\Omega)/c^2\partial t + \nabla.(U^2\beta\mathbf{K}) = 0$ (12)

These relations apply to progressive waves for $\beta=\pm 1$, to standing waves at rest for $\beta=0$ and in motion for $\beta<1$, to monochromatic waves for ω and $\mathbf{k}$ constant, to almost monochromatic waves for varying $\Omega(x,t)$ et$\mathbf{K}(x,t)$. They lead to dynamical properties for energy-momentum conservation, and to least action principle, for standing fields and almost standing fields [5-9].

For a standing wave with constant frequency ω, either at rest or in motion, (12) reduces to

$\partial u_0^2/\partial t_0 = 0.$ $\partial u^2/\partial t + \nabla.u^2\mathbf{v} = 0$

or

$\partial_\mu w^\mu = 0$ (13)

Where $w^\mu = (u^2, u^2\mathbf{v}/c) = u_0(x_0)^2(1, \mathbf{v}/c)/\sqrt{(1-\beta^2)}$ is a four-dimensional vector. This continuity equation for u^2 is formally identical with Newton's equation continuity for matter-momentum density μ

$\partial\mu/\partial t + \nabla.\mu\mathbf{v} = 0$ with $u^2 = \mu c^2$ (14)

By transposition, we can then admit that u^2 represents the energy density of the standing field.

In order to describe the kinematical behavior of a standing field, we may restrict to one of its particular point such as its center of amplitude, with position x_0 defined by (6), especially when experimental conditions lead us to consider that it may be reduced to a mass-point. The position x_0 of the energy density is such that

$\nabla_0 u_0^2 = 0$ $\nabla u^2 + (\partial u^2\mathbf{v}/c^2\partial t) = 0$ $\nabla\times\mathbf{v} = 0$

or

$\pi^{\mu\nu} = \partial^\mu w^\nu - \partial^\nu w^\mu = 0,$ (15)

The standing wave energy density u^2 is spread in space. It corresponds then to a potential energy density. In(15), $\mathbf{F} = -\nabla u^2 = -\nabla w_P =$ is a density force, $\partial u^2\mathbf{v}/c^2\partial t$ a density momentum, and $\pi^{\mu\nu}$ a four-dimensional force density.

In (15), the vanishing four-dimensional force density tensor $\pi^{\mu\nu}$ of a standing wave, asserts its space stability at rest remains in motion, and that the energy-momentum density four-vector w^μ is four-parallel, or directed along the motion velocity $\mathbf{v}$.

Equation (15) is mathematically equivalent to the least action relation, in which energy density w^μ is a four-dimensional gradient $\partial^\mu a$,

$\delta\int da = 0$ $\delta\int\partial^\mu a dx_\mu = 0$ with $w^\mu = \partial^\mu a.$ (16)

When we transpose the mass density $\mu = u^2/c^2$, and we take into account the identities $\nabla P^2 = 2(\mathbf{P}.\nabla)\mathbf{P} + 2\mathbf{P}\times(\nabla\times\mathbf{P})$ and $d\mathbf{P}/dt = \partial\mathbf{P}/\partial t + (\mathbf{v}.\nabla)\mathbf{P}$ forc and v constant, after integration with respect to space, we get the equation for matter

$$d\mathbf{p}/dt = -\nabla mc^2 + \{\nabla(mv)^2\}/2m \quad d\mathbf{p}/dt = \nabla L_m = -\nabla m_0c^2\sqrt{(1-\beta^2)}. \quad (17)$$

We retrieve the relativistic Lagrangian of mechanics for free matter $L_m = -m_0c^2\sqrt{(1-\beta^2)}$.

2.3. Electromagnetic Interaction

The continuity equation of an almost standing wave, is relating to the total energy density, $W = U^2\Omega = w + \delta W$, sum of the mean standing wave w and of the interactions δW. Relation (15) becomes

$$\Pi^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu = 0 \text{ or } \Pi^{\mu\nu} = \pi^{\mu\nu} + \delta\Pi^{\mu\nu} = 0 \quad (18)$$

By difference with the null four-dimensional force density $\pi^{\mu\nu}$ for a standing wave, only the total forcedensity $\Pi^{\mu\nu}$ for an almost standing wave vanishes. In the first case, this asserts the space stability of an isolated moving standing wave, while in the second case; the space stability concerns the whole almost standing wave. It behaves as a system composed of two sub-systems, the mean standing field with high frequency $\Omega(x,t)\approx\omega$, and the interaction field with lower frequency $\delta\Omega(x,t)$, each one exerting an equal and opposite non vanishing density force $\pi^{\mu\nu} = -\delta\Pi^{\mu\nu}$ against the other. The mean energy-momentum density tensor $\pi^{\mu\nu}$, no longer vanishes in (18), as previously in (15). This comes from the mean energy-momentum density four-vector w^μ, which is no longer parallel, because of the opposite density force $\delta\Pi^{\mu\nu}$ exerted by the interaction.

It appears that an almost standing field behaves as a whole system in motion which can be split in two sub-systems, the mean standing field and the interaction field. Both are moving with velocity v, while exerting each other opposite forces in different directions, including perpendicularly to the velocity v. The perturbation field, arising from local frequency variations $\delta\Omega(x,t)$, introduces orthogonal components in interaction density force and momentum.

After generalizing relations (17) by constants variation method for mass $M(x,t)=m\pm\delta M(x,t)$, we get

$$\nabla Mc^2+\partial P/\partial t=0 \quad \nabla\times P=0 dP/dt=-\nabla Mc^2+(\nabla P^2)/2M. \quad (19)$$

The force density $\delta\Pi^{\mu\nu}\neq 0$ exerted by the interaction is formally identical with the electromagnetic tensor $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu\neq 0$. We can set them in correspondence $\delta\Pi^{\mu\nu}= eF^{\mu\nu}$, through a constant invariant charge e, with $\delta M(x,t)= eV(x,t)/c^2$ and $\delta P(x,t)= eA(x,t)/c$. The double sign for mass variation corresponds to the two signs for electric charges, or to emission and absorption of electromagnetic energy by matter. We retrieve the minimum coupling of classical electrodynamics, $P^\mu(x,t)=p^\mu+eA^\mu(x,t)/c$, with $M(x,t)c^2=mc^2+eV(x,t)$, and $P(x,t)=p+eA(x,t)/c$, where electromagnetic energy exchanged with a particle is very small in comparison with its own energy $eA^\mu(x,t)/c= \delta P^\mu(x,t) << p^\mu$ [11]. Electromagnetic interaction is then directly linked to frequencies variations of the field ε.
Relation (19) yields the relativistic Newton's equation for charged matter with the Lorentz force

$$dP/dt= -\nabla m_0c^2\sqrt{(1-\beta^2)} + e(E+v\times H/c). \quad (20)$$

2.4 Adiabatic Invariant

The relation (11) leads to first order approximation for an almost standing wave,

$$[\partial U^2/\partial t+\mathbf{\nabla.}U^2\mathbf{v}]/U^2+\delta[\partial\Omega/\partial t+\mathbf{\nabla.}\Omega\mathbf{v}]/\Omega =0$$

or

$$(\partial_\nu W^\nu)/W+\delta(\partial_\nu\Omega^\nu)/\Omega =0 \quad (21)$$

with energy density $W=w\pm\delta W=\mu c^2=\mu c^2\pm\delta\mu c^2$, four-dimensional energy density $W^\nu=w^\nu\pm\delta W^\nu$, frequency $\Omega=\omega\pm\delta\Omega$, and four-dimensional frequency $\Omega^\nu =(\Omega,\Omega\mathbf{v}/c)$, leading to

$$W^\nu=I\Omega^\nu \quad \text{and} \quad \delta W^\nu=I\delta\Omega^\nu \quad (22),$$

when we take into account the double sign in frequency variation $\delta\Omega$. The constant I is an adiabatic invariant density.
Integrations with respect to space of μ and I densities, lead to relations between four-dimensional energy and four-dimensional frequency, through the adiabatic invariant H, formally identical with the Planck's constant h.$E^\nu=(mc^2,\mathbf{p}c)=m_0c^2u^\nu=H\omega^\nu=H(\omega,\mathbf{k}c)=H\omega_0u^\nu$ (23)
For the standing wave corresponding to matter, adiabatic variations of its frequency Ω lead to electromagnetic interaction constituted by progressive waves. The energy of electromagnetic interaction derives from mass variation $dE=c^2dm$. It leans directly upon the wave property of matter: its energy $dE=hd\nu= c^2dm$ derives from variations of matter energy $E=h\nu=mc^2$.

3. Applications to Maxwell's Equations

The Einstein's program provides a common framework which allows us to retrieve the main basic assumptions, and equations, of Maxwell's equations. Instead of admitting from the beginning the point-like particle as fundamental, in order to fit to experiment, it derives as a geometrical approximation of a continuous space-like field [12]. It is well known that carrying out an approximation leads to different final results, depending it occurs at the beginning, or at the end of a demonstration. In the first case, the neglected properties and data, hidden along the calculus process, yield a less complete result.

We propose to show how this applies to Maxwell's equations respectively to:
-the charge particle and electromagnetic field,
-the difference between both constant velocity of light c, with to the speed $v=\beta c<c$ of matter,
-the difference between kinematic behaviours of energy-momentum density four-vectors w^μ, which are four-parallel or directed along the motion velocity **v**for matter , and four-orthogonal for interaction,
-the difference between dynamical variables, for kinematical coordinates, and space variables for space repartition of mass or electric charge.

3.1 History

It is well known that, beyond the Maxwell's equations, electromagnetism plaid a fundamental role at origin of in light quantification by Planck in 1900 with introduction of h as universal constant, and at origin of special relativity by Einstein in 1905, with introduction the velocity of light in vacuum c, as universal constant.
One of the main successes of Maxwell's equations had been to show that the velocity c, which was of purely electromagnetic nature, since it derived from dielectric and magnetic permeability properties, was also the velocity of light. Afterwards, and especially after Hertz's experiments, light was included as a branch of electromagnetism. From a physical point of view, such integration was not too difficult, since before, the nature of light was not well specifically defined. This did not happen for matter, after Einstein admitted that " *the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good.1 We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies.* »[1].

In consequence, the relation $E=mc^2$, for equivalence between mass m and electromagnetic energy E, established a bridge, which has been verified in innumerable applications. Nowadays, the mass-energy of particles, even neutral, is usually expressed in electron-volts. Nevertheless mass is fundamentally independent from electric charge in the table of the Standard Model. Then, until now, matter has resisted to its integration within a unitary electromagnetic frame. On the contrary, its independence of Maxwell's equations led to deprive the speed of light c of its original electromagnetic nature. It became, either the speed of signal, and then associated to an abstract mathematical property, or more generally, the speed of all interactions, electromagnetic, gravitational, strong and weak. For the same reason, the Lorentz transformation, which determines the structure of Maxwell's equations in vacuum, has been furthered off its stationary wave physical sense, to describe geometrical properties of space-time coordinates of reference.

According to Einstein, *"The formulation of these equations is the most important event in physics since Newton's time, not only because of their wealth of content, but also because they form a pattern for a new type of law….The characteristic features of Maxwell's equations, appearing in all other equations of modern physics, are summarized in one sentence. Maxwell's equations are laws representing the structure of the field."* [2]. Nowadays, it still appears that*" One could believe that it would be possible to find a new and secure foundation for all physics upon the path which had been so successfully begun by Faraday and Maxwell. Accordingly, the revolution begun by the introduction of the field was by no means finished»* [3].

This emphasizes that Maxwell's equations, in which mass is explicitly absent, describe only electromagnetic properties of nature. The properties of matter arise indirectly when it is charged. Neutral matter is entirely out of its scope.

3.2 Charge Particle and Electromagnetic Field

The tridimensional formalism appears as well suited to exhibit physical properties of electromagnetic fields, such as the geometrical properties for vectors, either perpendicular $\nabla.\mathbf{A}_{\perp}=0$, orparallel $\nabla\times\mathbf{A}_{/\!/}=0$, and $\nabla a=\mathbf{A}_{/\!/}$, by comparison with the four-dimensional Minkowski's formalism.

It is well known that Maxwell's equations

$\nabla.\mathbf{E}=4\pi\rho$ $\nabla.\mathbf{H}=0$ $\nabla\times\mathbf{E}= -\partial\mathbf{H}/c\partial t$ $\nabla\times\mathbf{H}= \partial\mathbf{E}/c\partial t+ 4\pi\mathbf{j}/c$ (24)

derive from the Coulomb's force $F= kqq'/r^2$ between electric point charges q, q', yielding directly to the electric field $E= q/r^2$ and to $\nabla.\mathbf{E}= dE/dr= 4\pi dq/4\pi r^2\, dr= 4\pi\rho$. Such a derivation emphasizes that the charge density ρ involves a field variable r, which describes its extension in space. It becomes eluded by the point approximation for charge, and implicitly for matter supporting it. Thus, only dynamical variables remain in the Dirac's distribution $\rho_0(\mathbf{x}_0)= e\delta(\mathbf{x}_0)$. The constant velocity v of charge, due to supporting matter, determines the parallel direction for vectors and for the relativist space-time framework. This appears directly in the resulting continuity relation for constant moving charge $\partial\rho/\partial t+ \nabla.\mathbf{j}=0$, in which the current density $\mathbf{j}=\rho\mathbf{v}$ is necessarily parallel and the velocity **v** constant, so that $\nabla.\mathbf{j}= \mathbf{v}.\nabla\rho$. The continuity equation, identical with $d\rho/dt=0$, expresses that the charge density $\rho_0(x_0) \leftrightarrow \partial\rho_0(x_0)/\partial t_0 =0$, which is constant at rest, remains constant in uniform motion.

In order to illustrate the more complete result obtained when the point approximation is carried out at the end instead of at the beginning, the extended charge relations in moving and rest systems, deduced from (24)

$\nabla^2\mathbf{E}_{/\!/}= 4\pi\nabla\rho$ and $\nabla_0{}^2\mathbf{E}_{0/\!/}= 4\pi\nabla_0\rho_0$ (25)

Are static: they imply an instantaneous propagation of the Coulomb's field $\mathbf{E}_{/\!/}$. On another hand, when we add the relation in moving system, deduced from (24)

$-\partial^2\mathbf{E}_{/\!/}/c^2\partial t^2 = 4\pi\partial\mathbf{j}_{/\!/}/c^2\partial t$ (26)

we obtain

$\nabla^2\mathbf{E}_{/\!/} -\partial^2\mathbf{E}_{/\!/}/c^2\partial t^2= 4\pi(\nabla\rho +\partial\mathbf{j}_{/\!/}/c^2\partial t) \rightarrow \Box_0\ \mathbf{E}_{0/\!/} = 4\pi\nabla_0\rho_0$ (27)

This time, either in the moving or in the rest system, the Coulomb's field $\mathbf{E}_{/\!/}$ propagates at light velocity, in accord with special relativity. In addition, like in (6), the Cartezian position $\mathbf{x}_0$ of the charge, such that $\nabla_0\rho_0=0$ emphasizes that it behaves also as point source fort the exchanged field. The different velocities, c for the field $\mathbf{E}_{/\!/}$, and $v=\beta c<c$ for the charge density ρ, imply structural correspondences: the Coulomb's field with a progressive c-field $\mathbf{E}_{0/\!/}\leftrightarrow\varepsilon$in (1), and the charge density with a standing amplitude field $\rho_0\leftrightarrow u_0$ in (5).
In consequence, like for u_0, for which the constant character expresses only a mean result of a more complex perturbed amplitude field

$U(x_0,t_0)=u(x_0)\pm\delta U(x_0,t_0)$, with $\delta U(x_0,t_0)<< u(x_0)$ in (10).

We retrieve the structural dynamical behavior of electric charge in quantum electrodynamics, described by the Feynman's diagrams. It emits and absorbs continually virtual photons, or c-fields and electrons, or amplitude standing fields.

3.3. Maxwell's Equations and Matter

We notice that, although matter does not appear explicitly in Maxwell's equations, its implicit presence is necessary, as support of electric charges. It underlies the deduced charge conservation equation, in which the charge may be, either at rest, verifying the Coulomb's law, or in motion with a speed strictly inferior to, and different from, c. On another hand, electromagnetic fields as solutions propagate in vacuum at light velocity c. In classical framework, matter and electromagnetic field are physically independent: matter may exist without charge in Newton's or Einstein's equations, while electromagnetic field propagates in vacuum, and then without matter presence.

In Maxwell's equations, the distinction between matter and electromagnetic quantities involved is absolute. Matter is fundamentally different from electric charge. Nevertheless, it serves necessarily as support, fixing its velocity v, strictly inferior to the light c, which is the propagation velocity of the electromagnetic field **E,H**.

In 1905, when Einstein conjectured that neutral matter, like electrically charged, was relevant to the mathematical structure of Maxwell's equations, he established a bridge between mass m and electromagnetic energy E, with the relation $E=mc^2$. It has been verified in innumerable applications.

However, until now, matter remains well distinct from electromagnetism, even if in particles physics, the mass is expressed in electron-volts, independently of its electric charge. This is justified if we notice that, at world scale, the mass created from electromagnetic energy is negligible. At universe scale, one admits that whole matter was created from electromagnetic energy, once for all, at nucleosynthesis era, 13.8 billion years ago.

Inversely, neutral mass remains explicitly absent in Maxwell's equations. However, the Einstein's initial conjectures, and its following program, show that matter properties are contained implicitly.

When we neglect the oscillating field Coulomb's field in (27), we notice the formal identity with (15-17)

$$d\mathbf{j}_{/\!/}/dt= - c^2\nabla\rho+ (\nabla j^2)/2\rho$$

or

$$d\mathbf{j}_{/\!/}/dt= - (c^2 -v^2)\nabla\rho= =-\sqrt{(1-\beta^2)}\nabla\rho_0 c^2 \quad (28)$$

On another hand, instead of yielding the propagation of the electromagnetic field ($\mathbf{E}_\perp$, $\mathbf{H}_\perp$) in vacuum, by cancellation of the charge density ρ outside a localized particle, this no longer holds when, $\rho(x_0)$, like $u(x_0)$, is extended in space.

We notice then, that the continuity relation $\partial\rho/\partial t+ \nabla.\mathbf{j}=0$ admits as freely determined gauge, a perpendicular current density $\mathbf{j}_\perp$, independantly of the charge density ρ, since $\nabla.\mathbf{j}_\perp=0$.

$$\nabla.\mathbf{E}_\perp=0 \quad \nabla.\mathbf{H}_\perp=0 \quad \nabla\times\mathbf{E}_\perp= - \partial\mathbf{H}_\perp/c\partial t \quad \nabla\times\mathbf{H}_\perp= \partial\mathbf{E}_\perp/c\partial t+ 4\pi\mathbf{j}_\perp/c \quad (29)$$

Usually, such relations describe the electromagnetic field due to a point-like dipole formed by two opposite charges. But here, $\mathbf{j}_\perp(x,t)$ is extended in space-time, deriving from the perturbation field.

This emphasizes that Maxwell's equations, in which mass is explicitly absent, describe only electromagnetic properties of nature. The properties of matter arise indirectly when it is charged. Neutral matter is entirely out of its scope.

3.4. Quantification

It is usually admitted that the quantum Planck, as a fundamental constant, is extraneous to Maxwell's equations. However, we notice that it was introduced in order to describe the electromagnetic **radiation** emitted by a black body in thermal equilibrium at a definite temperature.

On one hand, Ehrenfest established the formal analogy of Planck's constant with an adiabatic invariant [13]. On another hand, L. de Broglie extended it application from electromagnetic radiation to mass-energy of an electron [14]. By comparison, two decades separate historically, the discovery of first quantification, for electromagnetic energy, from the second quantification, for matter. In addition, the particles of matter, such as electrons, and of interactions, such as photons, are entirely independent, physically and geometrically.

Since the c-scalar field of Einstein's program forms the underlying structure, not only for electromagnetic fields, but also for massive particles, it is implicit in Maxwell's equations. The relations (21-23) show that the Planck's constant corresponds formally to a phenomenological constant value of an adiabatic invariant field. A unique equation (21) gathers its application to standing fields, behaving like matter $E=h\nu=mc^2$, and to the perturbation field behaving like electromagnetic interaction $dE=hd\nu= c^2dm$.

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